

MATH 508 Foundations of Mathematics

Usual course content:

Logic: In this section we will look at logic as it is used in everyday mathematics and then at the role of logic in the foundation of mathematics. We will consider some non-traditional views of the role of logic in mathematics including the constructivist view of mathematics and its foundations.

The Axiomatic Method: We will consider the two principal ways that axiom systems are used. The first of these is to characterize a particular mathematical structure, the real numbers for example, by listing its properties. The second use of axiom systems is to describe a class of mathematical structures; for example, the class of groups is described in this way. We will look at several examples of axiom systems including some simple axioms for a kind of geometry and the Peano axioms for the natural numbers.

The Zermelo-Fraenkel Axiom System for Set Theory: These are the standard axioms for set theory that have been developed over the last 100 years. We will see how almost all of mathematics can be done with just these axioms as a starting point and therefore how they serve as a foundation for mathematics.

Relations, Functions, Infinite Sets and Cardinal Numbers: This is the beginning of the development of mathematics in set theory. We will study Georg Cantor's theory of sizes for infinite sets.

Well ordered sets and the axiom of choice: A continuation of the development of mathematics.

The construction of the number systems: We begin by constructing the natural numbers as sets; then from the natural numbers the other number systems are constructed.

Technology:

Varies by instructor

Students who may benefit:

Graduate students